

Spin Ordering and Quasiparticles in Spin Triplet Superconducting Liquids

Fei Zhou

ITP, Minnaert building, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands
(February 1, 2008)

Spin ordering and its effect on low energy quasiparticles in a p-wave superconducting liquid are investigated. We study the properties of a novel 2D p-wave superconducting liquid where the ground state is spin rotation invariant. In quantum spin disordered liquids, the low energy quasiparticles are bound states of the bare Bogolubov- De Gennes (*BdeG*) quasiparticles and zero energy skyrmions, which are charge neutral bosons at the low energy limit. Further more, spin collective excitations are fractionalized ones carrying a half spin and obeying fermionic statistics. In thermally spin disordered limits, the quasi-particles are bound states of bare *BdeG* quasi-particles. The latter situation can be realized in some layered p-wave superconductors where the spin-orbit coupling is weak.

Our fascination towards excitations carrying quantum numbers distinct from electrons in condensed matter systems dated back at least twenty years ago [1–4]. It is now widely accepted that there are a variety of strongly interacting systems where quasiparticles carry only a fraction of quantum numbers an electron has. All known examples, though have been discovered in very diversified condensed matter systems which bear no similarities at first sight, appear to share one remarkable common feature. It is topological excitations interacting with electrons one way or the other which make all sorts of exotic quasiparticles or collective excitations possible. This also lies in the heart of earlier examples discovered in field theories and mathematical physics [5–7].

In this letter, we scrutinize the spin ordering and its influences on excitations, particularly, on the fractionalization of quasiparticles and collective spin excitations in 2D spin triplet p-wave superconductors. Spin triplet superconducting states are believed to exist in ^3He , many heavy-fermion superconductors and most recently layer perovskite Sr_2RuO_4 crystals [8–11]. We report the existence of a new 2D spin triplet p-wave superconducting state which is spin rotation invariant. This new state is characterized by a finite range spin correlation and $hc/4e$ vortices as the elementary topological excitations. The elementary quasiparticles are Bogolubov-De Gennes (*BdeG*) quasiparticles hosted in zero energy skyrmions. The spin collective excitations are shown to be fractionalized ones carrying a half spin and obeying fermionic statistics, by contrast to the spin wave excitations in spin ordered p-wave superconducting states (*SOPSSs*). We should mention that the general fractionalization pattern in some p-wave superconductors was recently classified in [12].

For a p-wave superconductor with an order parameter $\mathbf{d}(\mathbf{k}) = \Delta_0(k_x + ik_y) \exp(i\chi)\mathbf{n}$ [8,9], the Hamiltonian in the Nambu space of $\Psi = (\psi^+, i\tau_2\psi)$ can be written as,

$$H = \sigma_3\epsilon + \sum_{i=x,y} \sigma_i \{\partial_i, \hat{\Delta}\}_+, \quad (1)$$

where $\hat{\Delta}$ is defined as $\hat{\Delta} = \Delta_0 \exp(i\sigma_3\chi)(\mathbf{n} \cdot \boldsymbol{\tau})$ and

$\epsilon(\mathbf{k}) = \hbar^2\mathbf{k}^2/2m - \epsilon_F$. We use σ as the Nambu space Pauli matrix and τ as the spin space one. We assume the spin-orbit interactions are weak and \mathbf{n} is a unit vector in a sphere S^2 . The internal space of the symmetry broken state is $\mathcal{R} = [S^1 \times S^2]/Z_2$. The order parameter observes a discrete symmetry: $\hat{\Delta}(\mathbf{n}, \chi) \rightarrow \hat{\Delta}(-\mathbf{n}, \chi + \pi)$ and represents a quantum spin nematic p-wave superconducting state.

Spin-phase separation To obtain an effective theory, we integrate over the fermionic degrees of freedom and make a gradient expansion. At low temperatures, we report the result as

$$\mathcal{L} = \mathcal{S}_{ab} \mathbf{P}_{sa} \mathbf{P}_{sb} - \mathcal{T}_0 \Phi^2 + \frac{1}{8\pi} (\mathbf{E}^2 + \mathbf{B}^2) + \mathcal{S}_{ab}^{\alpha\beta} \nabla_a \mathbf{n}_\alpha \nabla_b \mathbf{n}_\beta - \mathcal{T}^{\alpha\beta} \partial_t \mathbf{n}_\alpha \partial_t \mathbf{n}_\beta + \frac{\mathcal{N}}{4\pi} \epsilon^{\mu\nu\lambda} \mathbf{A}_\mu \mathbf{F}_{\nu\lambda}. \quad (2)$$

$\mathbf{P}_s = \frac{1}{2} \nabla \chi + e \mathbf{A}^{em}$ and $\Phi = \frac{1}{2} \partial_t \chi + e \phi^{em}$ are the gauge invariant momentum and potentials respectively. The superscript *em* is introduced to distinguish the usual electric magnetic vector potential \mathbf{A}^{em} from the topological field \mathbf{A} defined below in terms of \mathbf{n} . $\mathcal{T}_0 \nu_0^{-1}$ is a unity at $T = 0$ and varies smoothly as a function of the temperature, ν_0 is the averaged density of states at the fermi surface. $\mathbf{F}_{\mu\nu} = \frac{1}{2} \mathbf{n} \cdot \partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n}$, and \mathbf{A} is the vector potential of $\mathbf{F}_{\mu\nu}$. $\mathcal{S}_{ab}^{\alpha\beta} = \delta^{\alpha\beta} \mathcal{S}_{ab}$, $\mathcal{S}_{ab} = \rho_0/2m\tilde{\mathcal{S}}_{ab}$; and $\mathcal{T}^{\alpha\beta} = \delta^{\alpha\beta} \nu_0 \tilde{\mathcal{T}}$. Finally, $\tilde{\mathcal{S}}_{ab}$, $\tilde{\mathcal{T}}$ and \mathcal{N} are calculated as

$$\begin{aligned} \tilde{\mathcal{S}}_{ab} &= \frac{2m}{\rho_0} \int \frac{d^2k}{(2\pi)^2} \mathbf{v}_a \mathbf{v}_b F(\mathbf{k}), \quad \tilde{\mathcal{T}} = \frac{1}{\nu_0} \int \frac{d^2k}{(2\pi)^2} F(\mathbf{k}); \\ \mathcal{N} &= \int \frac{d^2k}{4\pi} \epsilon^{\alpha\beta\gamma} \frac{\mathbf{M}_\alpha}{\Delta_0^2} \frac{\mathbf{M}_\beta}{\partial k_x} \frac{\mathbf{M}_\gamma}{\partial k_y} F(\mathbf{k}); \\ F(\mathbf{k}, T) &= -\frac{\partial}{\partial \epsilon^2(\mathbf{k})} \frac{2\Delta_0^2(T)}{E(\mathbf{k})} \tan\left(\frac{E(\mathbf{k})}{2kT}\right). \end{aligned} \quad (3)$$

ρ_0 , m and ϵ_F are the density, mass and fermi energy respectively; $E(\mathbf{k}) = \sqrt{\epsilon^2(\mathbf{k}) + \Delta_0^2 \mathbf{k}^2/k_F^2}$, and Δ_0 is the temperature dependent gap. At zero temperature, \mathcal{N} is quantized to be unity in 2D. The director \mathbf{M} is defined as $\mathbf{M}(\mathbf{k}) = (v_\Delta \mathbf{k}_x, v_\Delta \mathbf{k}_y, \epsilon(\mathbf{k}))$; $v_\Delta = \Delta_0/k_F$.

When the director \mathbf{n} is a planar vector confined in the $\theta = \pi/2$ equator, i.e. $\mathbf{n} = (\cos \phi, \sin \phi, 0)$, the spatial gradient terms coincide with previous results for the A phase of ${}^3\text{He}$ [14,15]. The action is valid when the frequency and the wave vector are smaller than $\Delta_0(T)$ and $\xi_0^{-1}(T) = \Delta_0(T)/v_F$ respectively.

Eq.2 suggests a few important properties of the quantum spin nematic p-wave superconductors. First of all, the dynamics of spin \mathbf{n} and phase χ is completely decoupled at the low frequency limit (except the entanglement due to a Z_2 projection in the functional integral [12], which we will not discuss in this paper). It reflects spin-phase separation in a p-wave superconductor. Second, for an isotropic fermi surface which interests us in this article, $\mathcal{S}_{ab} = \delta_{ab}\rho_s(T)/2m$, and $\rho_s(T)$ is the temperature dependent superfluid density which vanishes at the critical temperature T_c . So the spin and phase dynamics are characterized by an $O(3)$ σ -model (NL σ M) and an xy model respectively. At the mean field approximation, $\mathbf{n} = \mathbf{e}_z$ and χ is a constant. This corresponds to a conventional $SOpSS$. There are three Goldstone modes; two of them are spin waves $\delta\mathbf{n} = (1, \pm i, 0)$ with a linear dispersion. In an isotropic case, $\tilde{\mathcal{S}}_{ab} = \delta_{ab}\tilde{\mathcal{S}}$; the spin wave velocity is $v_s(T) = v_F\sqrt{\tilde{\mathcal{S}}/2\pi\tilde{\mathcal{T}}}$. And the last mode is the usual plasma wave, with a dispersion $\omega = \sqrt{2\pi e^2\rho_0 k/m}$ in 2D at $T = 0$.

The topological term was previously derived in [16,17]. This term, originating from the broken time reversal and parity symmetries, determines the topological order in the fields $F_{\mu\nu}$ and defines the structure of quasiparticles. Implications of topological terms in other unconventional superconductors were also explored recently [18–20]. Here, we investigate the spin ordering, zero energy spin textures and quasiparticles based on Eq.2. Let us emphasize that Eq.2 is valid as far as the quasiparticles are gapped and the gradient expansion is possible; physically, it says all low lying collective excitations below the BCS energy gap are correctly described by the action.

Spin ordering Because of an extra branch of Goldstone modes in the spin sector, the spin order is more fragile than the phase order in the problem. In 2D, this provides a unique possibility of spin disordered p-wave superconducting states ($SDpSSs$) where the S^2 -symmetry is restored and only the $U(1)$ -symmetry is broken. Such a state which is rotation invariant in nature differs from the conventional $SOpSSs$ where the S^2 symmetry is broken and there is a long range order in \mathbf{n} .

The finite temperature phase diagrams of the $O(3)$ Nonlinear σ Model (NL σ M) were previously analyzed in great details [21]. In the current situation, just as the superfluid velocity $\rho_s(T)$, all coefficients in the action, $\mathcal{S}_{ab}^{\alpha\beta}, \mathcal{T}_{ab}, \mathcal{S}_{ab}, \mathcal{T}$ depend on temperatures because of quasiparticle excitations. Taking this into account, we arrive at the following results in 2D.

When $\Delta_0 \ll \epsilon_F$, the spin order is established at zero

temperature and the correlation length

$$\xi_2 = \frac{v_s(T)}{\Delta_s(T)}, \Delta_s = T \exp\left(-\frac{2\pi[\rho_s(T)/2m - \Gamma]}{T}\right) \quad (4)$$

is finite only at finite temperatures (in a saddle point approximation). Here $\Gamma \sim \Delta_0(T)$. For most of p-wave superconductors, the gap energy is about $1K$ and the Fermi energy of order $1eV$, the superconductors are spin ordered at zero temperature. However, the liquids are spin disordered (in the absence of spin-orbit coupling) at any finite temperature as shown in Eq.4. On the other hand, when Δ_0 is much larger than the fermi energy, the spin long range order could be spoiled by quantum fluctuations and the rotation invariance is preserved [22]. We will be concerned with both situations, *thermal* and *quantum 2D SDpSSs* in the following discussion.

Zero energy skyrmions One of the most important feature of the *quantum SDpSS* is the existence of topological order and consequently topological stable zero energy skyrmions in the absence of spin stiffness. In $(2+1)$ space $\mathbf{x} = (\tau, \mathbf{r})$, it is convenient to introduce a field, $\mathbf{H}_\eta = \frac{1}{2}\epsilon^{\eta\mu\nu}\mathbf{F}_{\mu\nu}$. $\mathbf{H}_\tau = \mathbf{F}_{xy}$ represents $U(1)$ magnetic fields along z direction, $\mathbf{H}_x = \mathbf{F}_{y\tau}$ and $\mathbf{H}_y = \mathbf{F}_{\tau x}$ are the x, y -components of the electric field. To facilitate a calculation at finite temperatures, the perimeter along τ -direction L_τ is taken to be finite, i.e. $L_\tau = (kT)^{-1}$. Consider a rotating skyrmion terminated at the origin in a $S^2 \times S^1$ space $\mathbf{n}(\rho, \phi) = (\sin \theta(\rho) \cos(\tilde{\phi}), \sin \theta(\rho) \sin(\tilde{\phi}), \cos \theta(\rho))$ where

$$\begin{aligned} \tilde{\phi} &= Q_m \phi - \gamma(\tau), \\ \theta(\rho, \tau) &= 2\arccos \frac{\rho}{\sqrt{\rho^2 + v_s^2 \tau^2}} \Theta(\tau), \\ \gamma(\tau + L_\tau) - \gamma(\tau) &= N2\pi. \end{aligned} \quad (5)$$

One can confirm that $\nabla \cdot \mathbf{H} = Q_m 2\pi \delta(\tau) \delta(\mathbf{r})$, corresponding to a space-time monopole of charge Q_m in $2+1d$. As ρ, τ approach infinity, $\mathbf{H}(\rho, \tau)$ becomes vanishingly small. The action of this Euclidean space monopole event is finite ($a \sim 1$),

$$\mathcal{S}_m = \frac{a\Delta_0}{16\pi\Delta_s} + i\gamma_B, \gamma_B = \frac{Q_m \mathcal{N}}{4} [\gamma(L_\tau) - \gamma(\tau_0)]. \quad (6)$$

However, it has a Berry's phase due to the topological term, which characterizes a rotation of the skyrmion during its duration. $\gamma_B(0)$ obviously depends on the temporal coordinate at which the skyrmion is terminated, leading to destructive interferences between monopoles centered at different τ_0 with different rotation angles γ_B .

As a result, the fluctuations of space-time monopole events per unit volume are ($c \sim 1$)

$$\langle Q_m^2 \rangle = \delta(\mathcal{N}) \frac{\Delta_0}{c\xi_0^2} \exp\left(-\frac{a\Delta_0}{16\pi\Delta_s}\right). \quad (7)$$

Eq.7 shows that at any finite \mathcal{N} all monopole events are suppressed due to destructive interferences. It also implies that for $\mathcal{N} \neq 0$ the ground state has an infinite-fold degeneracy compared with that of $\mathcal{N} = 0$.

There are at least two important intraconnected consequences of the destructive interferences. First, Eq.7 indicates the conservation of the Skyrmon charges at $\mathcal{N} \neq 0$ in a *quantum SDpSS*, that is in the absence of the spin rigidity. If we define $c_w(\{\mathbf{n}(\mathbf{r})\}) = \frac{1}{2\pi} \int dx dy \mathbf{H}_z$ as the total number of Skyrmons living on the 2D sheet, in the presence of space-time monopoles Q_m at $\{\mathbf{r}^m, \tau_m\}$,

$$\frac{\partial c_w(\tau)}{\partial \tau} = \sum Q_m \delta(\tau - \tau_m). \quad (8)$$

A space-time monopole essentially connects a trivial vacuum to a Skyrmon configuration and causes a change in the topological charge c_w by one unit. At $\mathcal{N} = 1$, following Eqs.8, 7, we conclude that a skyrmion whose energy could vanish in the absence of the spin stiffness, is a well-defined topological configuration in a *quantum SDpSS*. This remarkable feature which doesn't exist at $\mathcal{N} = 0$ is also a consequence of a zero energy fermionic mode hosted by instantons.

Second, the suppression of monopole events leads to very distinct behaviors of fields $\mathbf{F}_{\mu\nu}$ in *SDpSSs*. The Wilson-loop integral defined as $\mathcal{W}_{U(1)} = \langle \mathcal{P} \exp(i \oint \mathbf{A} \cdot d\mathbf{r}) \rangle$ has different asymptotical behaviors in the large loop limit in the presence or absence of topological order in c_w . When the topological charge c_w is conserved at any finite \mathcal{N} , $\mathcal{W}_{U(1)} = \exp(-L_c C_1)$ (L_c is the perimeter of the Wilson loop) and the gauge fields are deconfining. This is true for a *quantum SDpSS* at zero and finite temperatures as far as L_τ is longer than the duration of space-time monopoles. However, in a *thermal SDpSS*, c_w is unconserved and the gauge fields are confining (except around the quantum critical point which I will not discuss here).

Quasi-particles We now employ the generalized Bogolubov-De Gennes equation to study the properties of quasiparticles in *SDpSSs*. In the presence of a topological configuration of $\mathbf{n}(\mathbf{r})$, it is convenient to introduce a gauge transformation $\Psi \rightarrow U_s(\mathbf{n})U_c(\chi)\Psi$ and work in a rotated representation; then one obtains a new Hamiltonian

$$H = \sigma_3 \epsilon(i\hat{\nabla}) + v_\Delta \sum_{i=1,2} \{\sigma_i \tau_3, i\hat{\nabla}_i\}_+. \quad (9)$$

Here $v_\Delta = \Delta_0/k_F$, $i\hat{\nabla} = i\nabla - \mathbf{A}_c - \mathbf{A}_s$ is a covariant derivative. We have defined $U_s^{-1} \mathbf{d} \cdot \tau U_s = \tau_3$, $U_c^{-1} \sigma_i \exp(i\sigma_3 \chi) U_c = \sigma_i$. The vector potentials are defined in terms of the $U(1)$ rotation U_c and $SU(2)$ rotation U_s as $\mathbf{A}_{c\mu} = iU_c^{-1} \partial_\mu U_c = \sigma_3(\mathbf{A}_\mu^{em} + \frac{1}{2} \partial_\mu \chi)$, $\mathbf{A}_{s\mu} = iU_s^{-1} \partial_\mu U_s = \tau_\alpha \cdot \mathbf{W}_\mu^\alpha$ ($\mu = 0, 1, 2$, stands for coordinates in 1+2 dimension space.). An explicit calculation also shows that $\mathbf{W}_\mu^3 = \mathbf{A}_\mu$. At last, the corresponding Lagrangian density is

$$\mathcal{L}_{BdeG} = \Psi^\dagger (\hat{\partial}_\tau - \mathcal{H}(\mathbf{A}_\mu^{em}, \mathbf{A}_{s\mu})) \Psi, \quad (10)$$

and $\hat{\partial}_\tau = \partial_\tau - \sigma_3 A_0^{em} - \tau \cdot \mathbf{A}_{s0}$.

Following Eq.9, besides a usual electric magnetic charge defined with respect to $\mathbf{A}_{e.m.}$ fields, a *BdeG* quasiparticle also carries a unit $U(1)$ -charge with respect to \mathbf{A}_μ fields and is minimally coupled with $\mathbf{F}_{\mu\nu}$. The energy of a *BdeG* particle is determined by the Wilson loop integral of \mathbf{A} . In *quantum SDpSSs*, the Wilson loop integral decays exponentially as a function of the perimeter of the loop. The interactions between *BdeG* quasiparticles mediated by the topological fields $\mathbf{F}_{\mu\nu}$ are rather weak and the *BdeG* quasiparticle energy is finite. But most importantly, in this case, skyrmions themselves carry $U(1)$ charges with respect to the fields \mathbf{A}_μ . This is indicated in Eq.2 if we introduce the skyrmion density-current density as $4\pi \mathbf{j}_\mu = \mathcal{N} \epsilon_{\mu\nu\eta} \partial_\nu \mathbf{A}_\eta$ and express the topological term in a form of minimal coupling $\mathbf{j}_\mu \mathbf{A}_\mu$. By minimizing the action of $\mathcal{L} + \mathcal{L}_{BdeG}$ with respect to \mathbf{A}_0 , \mathbf{A}^{em} and taking $\mathcal{N} = 1$ at low temperature limit, we do obtain a saddle point equation $4\pi < \Psi^\dagger \tau_3 \Psi > = \mathbf{e}_z \nabla \times \mathbf{A}$, $< \Psi^\dagger \sigma_3 \Psi > = 0$. This indicates that a skyrmion configuration carries a half spin but no charge. In other words, a spin $\frac{1}{2}$ but chargeless *BdeG* quasiparticle is hosted by, or confined with a skyrmion, with the confinement mediated by the spin fluctuations.

To examine the *BdeG* quasiparticles dressed with spin textures, we consider a skyrmion in polar coordinates (ρ, ϕ) . The director has a spatial distribution as $\mathbf{n}(\rho, \phi) = (\sin \theta(\rho) \sin \phi, \sin \theta(\rho) \cos \phi, \cos \theta(\rho))$; $\theta(\rho)$ is a smooth function of ρ , the asymptotics of which is $\theta(\rho = 0) = 0$ and $\theta(\rho \rightarrow \infty) = \pi$. The corresponding \mathbf{A} field can be chosen as

$$\mathbf{A} = \frac{1 - \cos \theta(\rho)}{2\rho \sin \theta} \mathbf{e}_\phi, \nabla \times \mathbf{A} = \frac{\sin \theta(\rho)}{2\rho} \frac{\partial \theta(\rho)}{\partial \rho} \mathbf{e}_z. \quad (11)$$

The $SU(2)$ field \mathbf{W}^α at $\rho \rightarrow \infty$ can be shown to take a simple form; $\mathbf{W}_i^3 = \mathbf{A}_i$ ($i = 1, 2$), $\mathbf{W}_0^3 = 0$ and $\mathbf{W}_\mu^1 = \mathbf{W}_\mu^2 = 0$.

A *BdeG* quasiparticle remains gapped in a texture. However, following Eq.11 when a spin-1/2 *BdeG* particle moves in a closed circle of radius ρ in a skyrmion defect, it acquires a Berry's phase of $\pi[1 - \cos \theta(\rho)]$ which approaches 2π at an infinity ρ . Consequently, under the interchange of coordinates, the two-body wave function of composite quasiparticles acquires an additional π phase because of hosting skyrmions, and $\Psi(\mathbf{r}_1, \mathbf{r}_2) = \Psi(\mathbf{r}_2, \mathbf{r}_1)$, which also follows the linking number theorem for skyrmions [23]. These composite quasi-particles are therefore Bosons. We also observe the *BdeG* quasiparticles are charge neutral at $\epsilon(\mathbf{k}) = 0$ with respect to an *em* field; they also carry zero $U(1)$ charges so to minimize the interaction between composite excitations. The life time of the quasiparticles is limited by the life time of

zero energy skyrmions; for the *quantum* disordered case, the zero energy skyrmions are stable even at low temperatures.

In *thermal SDpSSs*, the suppression of space-time monopoles is incomplete since L_τ is longer than the monopoles' duration Δ_s^{-1} . The gauge field then is confining and the *BdeG* quasi-particles form bound states, with zero or one total spin. This unexpected feature should be observed in future experiments on some layered p-wave superconductors.

Collective spin excitations The nature of the collective spin excitations in an *SDpSS* can be explored in a spinor representation of Eq.2. By introducing $\eta^+ \tau \eta = \mathbf{n}$, $\eta = (\eta_1, \eta_2)^T$ and $\eta^+ \eta = 1$, we obtain for η the following Lagrangian in *SDpSSs*,

$$\mathcal{L}_\eta = \frac{1}{2f^2} |(i\partial_\mu - \mathbf{A}_\mu)\eta|^2 + \frac{\Delta_s(T)}{\Delta_0(T)} \eta^+ \eta + \frac{\mathcal{N}}{4\pi} \epsilon^{\mu\nu\lambda} \mathbf{A}_\mu \mathbf{F}_{\nu\lambda}. \quad (12)$$

And η is a bosonic field carrying a unit charge with respect to \mathbf{A} fields and spin 1/2. In Eq.12, $2f^2 = 2m\Delta_0/\sqrt{\tilde{S}\tilde{T}}\rho_0$; we have introduced the following rescaling: $t \rightarrow t\xi_0/v_s$, $\mathbf{r} \rightarrow \mathbf{r}\xi_0$.

In quantum *SDpSSs*, an η - quantum is bound with a skyrmion such that the bound state becomes a fermion [24]. Each spin one spin wave excitation which is an elementary excitation in an *SOpSS*, is fractionalized into two elementary fermionic spinors hosted in skyrmions in quantum *SDpSSs*. Each spinor-skyrmion composite is a spin-1/2 excitation carrying no $U(1)$ charge, by contrast to a bare η excitation. In the thermal *SDpSSs*, the spin collective excitations are spin-wave ones with spin one.

$\frac{hc}{4e}$ vortices For the sake of completeness, I am also listing some properties of vortices. The linear defects in a symmetry broken state with an internal space $\mathcal{R} = [S^1 \times S^2]/Z_2$ have been recently discussed extensively in the context of Bose-Einstein condensates of ^{23}Na [13]. In *SOpSSs*, the linear defects are superpositions of $hc/4e$ vortices and π -disclinations because of the Z_2 symmetries in the problem. And a bare $hc/4e$ vortex is forbidden because of the catastrophe of a cut. In *SDpSSs*, however, $hc/4e$ vortices can exist by their own right and are elementary excitations.

The *SDpSS* discussed here has the following order parameters: $\langle \hat{\Delta} \rangle = 0, Tr \langle \hat{\Delta} \hat{\Delta} \rangle \neq 0, \langle \exp(i\chi) \rangle \neq 0$. The existence of SC^* with $\langle \hat{\Delta} \rangle = 0, Tr \langle \hat{\Delta} \hat{\Delta} \rangle \neq 0, \langle \exp(i\chi) \rangle = 0$, and other fractionalized states examined in [12] appears to be beyond the model studied here. Physically, the *SDpSS* has Josephson oscillations of $2eV$ frequency while in SC^* the frequency is $4eV$.

In the presence of spin-orbital couplings, the mean field solution indicates that the director of \mathbf{n} points along $\pm \mathbf{e}_z$ direction and the internal space is $[Z_2 \times S^1]/Z_2$. However, at an energy scale higher than the spin-orbit coupling ones, \mathbf{n} would be free to rotate on a two-sphere.

The spin order-disorder transition still takes place at a finite temperature below the superconductor-metal transition temperature T_c when the spin-orbit scattering rate is much smaller than $\Delta_0(0)$. As the spin is disordered, the above discussions on the spin textures and *BdeG* quasi-particles are still valid.

In conclusion, we also would like to remark that some aspects of the *BdeG* quasiparticles in spin disordered superconductors considered here reminisce the chiral-bag defect model for the nucleon [25,26]. The presence of spin-1/2 bosonic chargeless *BdeG* excitations in a quantum *SDpSS* is an example of fermi number fractionalization; it belongs to the same class phenomenon as the mid-gap quasiparticles hosted in domain wall excitations in one dimension polyacetylene [1] and the statistical transmutation proposed in some magnetic models [24,28,27,29]. It is my pleasure to thank P. W. Anderson, E. Demler, D.Khmelnitskii, K. Schoutens and F. Wilczek for useful discussions, and X. G. Wen for a conversation on instantonic zero modes. Finally, I am grateful to P.Wiegmann for explaining to me the global anomalies and patiently scrutinizing my arguments.

-
- [1] W. P. Su, J. R. Schrieffer and A. J. Heeger, Phys. Rev. Lett. **42**, 1698(1979).
 - [2] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395(1983).
 - [3] P. W. Anderson, Science **235**, 1196(1987).
 - [4] G. Baskaran, Z. Zou and P. W. Anderson, Solid State Commun. **63**, 873(1987).
 - [5] R. Jackiw and C. Rebbi, Phys. Rev. **D 13**, 3398(1976).
 - [6] G. 't Hooft and P. Hasenfratz, Phys. Rev. Lett. **36**, 1119(1976); J. Jackiw and C. Rebbi, Phys. Rev. Lett. **36**, 1116(1976).
 - [7] J. Goldstone and F. Wilczek, Phys. Rev. Lett. **47**, 968(1981).
 - [8] D. Vollardt and P. Wolfle, *The superfluid phases of ^3He* , Taylor and Francis, New York (1990).
 - [9] G. E. Volovik, *Exotic properties of superfluid ^3He* , World Scientific, Singapore (1992).
 - [10] R. Heffner and M. Norman, Com. Cond. Mat. Phys. **17**, 361(1996).
 - [11] Y. Maeno et al., Nature **372**, 532(1994); K. Ishida et al., Nature **396**, 658(1998); G. M. Luke et al., Nature **394**, 558(1998). See also T.M. Rice and M. Sigrist, J. Phys.: Condens. Matter **7**, L643(1995).
 - [12] E. Demler et al., cond-mat/0105446. See also [13] for discussions on bosonic quantum spin nematic states.
 - [13] F. Zhou, Phys. Rev. Lett. **87**, 080401-1(2001); E. Demler and F. Zhou, cond-mat/0104409; F. Zhou, cond-mat/0108473(2001).
 - [14] P. W. Anderson, *Basic Notions of Condensed matter Physics*, Benjamin and Cummings, Menlo Park, California (1984).

- [15] M. C. Cross, J. Low. Temp. Physics. **21**, 525(1975).
- [16] G. E. Volovik and V. Yakovenko, J. Phys: Cond. Matt. **1**, 5263(1989); We have omitted a topological term in the phase sector.
- [17] G. E. Volovik, Zh. Eksp. Teor. Fiz. **51**, 111(1990)[JETP Lett. **51**, 125(1990)].
- [18] T. Senthil, J. Marston and M. P. Fisher, Phys. Rev. **B 60**, 4245(1999).
- [19] J. Goryo and K. Ishikawa, Phys. Lett. **A 260**, 294(1999).
- [20] N. Read and D. Green, Phys. Rev. **B61**, 10267(2000).
- [21] S. Chakravarty et al., Phys. Rev. Lett. **60**, 1057(1988); Phys. Rev. **B 39**, 2344(1989).
- [22] This is supported by a saddle point calculation.
- [23] F. Wilczek and A. Zee, Phys. Rev. Lett. **51**, 2250(1983).
- [24] I. Dzyaloshinskii et al., Phys. Lett. **127 A**, 112(1988).
- [25] T. H. R. Skyrme, Proc. Roy. Soc. London **A260**, 127(1961).
- [26] J. Goldstone and R. L. Jaffe, Phys. Rev. Lett. **51**, 1518(1983).
- [27] S. A. Kivelson et al., Phys. Rev. **B35**, 8865(1987).
- [28] P. B. Wiegmann, Phys. Rev. Lett. **60**, 821(1988).
- [29] X.G. Wen, F. Wilczek and A. Zee, Phys. Rev. **B 39**, 11413(1990).